

APPLICATION NO. 09/826,118

TITLE OF INVENTION: Wavelet Multi-Resolution Waveforms

INVENTOR: Urbain A. von der Embse

Currently amended CLAIMS

APPLICATION NO. 09/829,118

INVENTION: Multi-Resolution Waveforms

INVENTORS: Urbain A. von der Embse

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CLAIMS

10 WHAT IS CLAIMED IS:

Claim 1. (deleted)

Claim 2. (deleted)

Claim 4. (deleted)

15 Claim 5. (deleted_

Claim 6. (deleted)

Claim 7. (new) A method for implementing nother Wavelet
waveforms and filters for communication applications, Aniterative
20 eigenvalue least squares LS digital mother Wavelets at baseband
for means for the design of new multi-resolution waveforms and
filters, said method comprising steps:
said mother Wavelet $\psi(n)$ with sample index n is a digital finite
impulse response (FIR) waveform at baseband (zero frequency
25 offset) in the time domain,
requirements for linear phase FIR filters are specified by the
passband and stopband performance of the power spectral
density (PSD) which requirements are incorporated into
quadratic error metrics $J(\text{pass})$, $J(\text{stop})$,
30 power spectral density PSD representative requirements for said
mother Wavelet ψ frequency ω response $\psi_{\omega}(\omega)$ in a multi-
channel filter bank, specify
a) passband frequency range for waveform transmission,
b) stopband spacing between adjacent filters,
35 c) bounds on ripple over said passband,

- ~~d) stopband filter attenuation,~~
- ~~e) rolloff with frequency outside stopband,~~
- ~~f) quadrature mirror filters QMF require the sum of said~~
~~PSD's for contiguous filter responses to be flat over~~
- 5 ~~deadband which is said stopband,~~
- ~~g) symbol to symbol interference ISI,~~
- ~~h) adjacent channel interference ACI,~~

~~said LS error metrics to measure said requirements (a) - (h) are~~
~~derived as functions of said Wavelet $\psi(n)$ assuming~~

- 10 ~~i) T and 1/T are sample interval and sample rate equal to~~
~~Nyquist sample rate,~~

~~j) ψ is real and symmetric about $n=0$,~~

~~k) $n=0, \pm 1, \dots, \pm ML/2$ digital index over said ψ ,~~

~~l) M is interval between contiguous said ψ ,~~

- 15 ~~m) 1/MT is said ψ symbol rate and channel to channel~~
~~separation,~~

~~n) L is length of said ψ in units of said M,~~

~~said multiple resolution properties require said LS metrics~~
~~to be constructed as functions of said Wavelet Fourier~~

- 20 ~~harmonics $\psi_k(k)$ with $k=0, \pm 1, \dots, \pm (N_k-1)$ and~~

~~$N_k \geq L$ is a design parameter,~~

~~it is sufficient to use positive $n=0, 1, \dots, ML/2$ and $k=0, 1, \dots$~~

~~$, N_k-1$ since said $\psi(n)$ and $\psi_k(k)$ are real and symmetric,~~

~~$ML/2+1 \times N_k$ matrix bw wherein "x" reads "by" maps $\psi_k(k)$ into~~

- 25 ~~$\psi(n)$ to within a scale factor by equations~~

~~$\psi(n) = \sum_k bw(n+1, k+1) \psi_k(k)$ for $n \geq 0, k \geq 0,$~~

~~$bw(n+1, k+1) = 1$ for $n=0,$~~

~~$= 2 \cos(2\pi nk/ML)$ otherwise,~~

~~$=$ row $n+1$, column $k+1$ element of bw ,~~

- 30 Wavelet requirements on the deadband for quadrature mirror
filter (QMF) properties for perfect reconstruction are

incorporated into the quadratic error metric $J(\text{dead})$,

Wavelet orthogonality requirements are expressed by the error

metrics $J(\text{ISI})$, $J(\text{ACI})$ for intersymbol interference (ISI)
and adjacent channel interference (ACI) which are non-
linear quadratic error metrics in said FIR $\psi(n)$ used to
control said ISI and ACI levels,

5 Wavelet multi-resolution property requires said quadratic error
metrics to be converted to quadratic error metrics in the
discrete Fourier harmonics $\psi(k)$ of said $\psi(n)$ wherein k is
the harmonic index,

10 eigenvalue algorithm requires the error metrics to be linear
quadratic forms in the $\psi(k)$ and finds the eigenvectors
equal to the $\psi(k)$ coefficients which minimize the weighted
sum J of said quadratic error metrics,

step 1 of the iterative algorithm implements said eigenvalue
algorithm to find said optimum $\psi(k)$ for the weighted sum
15 sum of $J(\text{pass})$, $J(\text{stop})$, $J(\text{dead})$,

step 2 linearizes said $J(\text{ISI})$, $J(\text{ACI})$ with said $\psi(k)$ from step 1,
step 3 uses said eigenvalue algorithm to find said optimum $\psi(k)$
for the sum J of $J(\text{pass})$, $J(\text{stop})$, $J(\text{dead})$ and the
linearized $J(\text{ISI})$, $J(\text{ACI})$,

20 step 4 checks to see if said iteration has converged,
step 5 returns to step 2 if said iteration has not converged and
linearizes said $J(\text{ISI})$, $J(\text{ACI})$ with said $\psi(k)$ from step 4,
and stops iteration if said iteration converges,

said $\psi(k)$ from said iteration algorithm is the optimum least-
25 squares error solution that minimizes said J ,

use inverse discrete Fourier transform of said $\psi(k)$ to calculate
 $\psi(n)$ which minimizes J ,

use said $\psi(n)$ for the transmitted data symbol waveform in the
communications transmitter and,

30 use complex conjugate of said $\psi(n)$ for the impulse response of
the detection filter in the communications receiver to
remove the received $\psi(n)$ and recover said transmitted data
symbols.

~~said LS error metrics are converted by said bw mapping into quadratic forms in said h_k equal to $J(\text{band}) = h_k' R h_k$ wherein said h_k' is the transpose of h_k and said R is a real square symmetric matrix of LS errors in meeting said requirements,~~
5 ~~LS ISI, ACI error metrics $J(\text{ISI}), J(\text{ACI})$ are derived as non-linear quadratic forms in h and converted by said bw matrix to the non-linear quadratic form in h_k equal to $J(\text{ISI}) = \delta E' \delta E$, $J(\text{ACI}) = 2\delta E' \delta E$ wherein $\delta E = A H h_k$ is a column vector and matrix "A" in the matrix product AH is a function of said h~~
10 ~~hereby introducing said non-linearity, and said AH differ for ISI and ACI error metrics,~~
~~LS cost function J is the weighted sum of said LS error metrics~~
~~————— $J = \sum w(\text{LS metric}) J(\text{LS metric})$~~
~~with summation over said LS metrics= passband, stopband,~~
15 ~~QMF deadband, ISI, ACI with normalized weights~~
 ~~$\sum w(\text{LS metric}) = 1$,~~
~~said weights are free design parameters,~~
~~said iterative eigenvalue LS algorithm at each step finds the~~
~~———— optimum eigenvalue and eigenvector which minimize said~~
20 ~~quadratic form J in h_k for a constant said "A",~~
~~said eigenvector is the optimum h_k which minimizes said J and~~
~~———— said bw equation derives the corresponding optimum h which~~
~~———— minimizes said J ,~~
~~step 1 in said iterative algorithm finds said optimum eigenvalue,~~
25 ~~eigenvector, h_k , h of J reduced by deleting said non-linear~~
~~ISI and ACI LS quadratic error metrics,~~
~~said h is used to evaluate said "A" matrices for step 2,~~
~~step 2 finds said optimum eigenvalue, eigenvector, h_k , h for~~
~~———— minimum J using said "A" from step 1,~~
30 ~~said h is used to evaluate said "A" for step 3,~~
~~steps 3, 4, etc. continue until said minimum J converges to a~~
~~———— steady value and,~~
~~said optimum $\psi_k(k)$ uses said bw to calculate optimum $\psi(n)$ for~~
~~———— implementation as said Wavelet FIR digital waveform and~~

~~filter time response.~~

Claim 8. (~~new~~currently amended) A second method for
5 implementing mother Wavelet waveforms and filters for
communication applications, An LS method for designing digital
mother Wavelets at baseband for multi-resolution waveforms and
filters, said method comprising steps:
construct said error metrics $J(\text{pass}), J(\text{stop}), J(\text{dead}), J(\text{ISI}),$
10 $J(\text{ACI})$ as quadratic error metrics in $\psi(k)$ as depicted in
claim 7 and convert these quadratic forms to norm-squared
error metrics in $\psi(k)$ for least-squares gradient solution
and construct J as their weighted sum,
step 1 calculates an initial estimate $\psi(k)$ of said solution using
15 the Remez-Exchange algorithm for the weighed sum of
 $J(\text{pass}), J(\text{stop})$ represented as quadratic error metrics in
 $\psi(k),$
~~said PSD waveform representative requirements and assumptions~~
~~are recited in (a) (n) in Claim 7,~~
20 ~~said multiple-resolution properties require said LS metrics~~
~~to be constructed as functions of said $\psi_k(k),$~~
~~said LS error metrics for said passband, stopband, and QMF~~
~~deadband requirements are derived as squared vector norm~~
~~functions of said h and converted by said bw matrix into~~
25 ~~$J(\text{band}) = \|Bh_k\|^2$ wherein $\|Bh_k\|$ is the vector norm of the~~
~~column vector Bh_k and said B is the matrix of LS errors in~~
~~meeting said requirements and wherein said squared vector~~
~~norm is suitable for LS optimization,~~
~~LS ISI, ACI error metrics $J(\text{ISI}), J(\text{ACI})$ are derived as squared~~
30 ~~vector norm functions equal to $J(\text{ISI}) = \|\delta E\|^2, J(\text{ACI}) = 2\|\delta E\|^2$~~
~~using said column vectors $\delta E = Ah_k$ in claim 7,~~
~~LS cost function J is said weighted sum of said LS error metrics~~

~~equal to $J = \sum w(\text{LS metric}) J(\text{LS metric})$ defined in claim 7,~~
~~an LS gradient search algorithm finds optimum $h_k(k)$ to~~
~~minimize J ,~~
~~step 1 of said LS gradient search algorithm uses a Remez-~~
5 ~~exchange algorithm to find said optimum $h_k(k)$ for said J~~
~~reduced to said passband and stopband LS metrics,~~
~~step 2 uses the estimated $h_k(k)$ from step 1 to initialize said~~
~~gradient search,~~
~~step 2 uses said estimate $\psi(k)$ from step 1 to initialize said~~
10 ~~gradient algorithm,~~
~~step 3 selects one of several available gradient search~~
~~algorithms, gradient search parameters, and stopping rules,~~
~~step 4 implements said algorithm, parameters, and stopping rule~~
~~selected in step 3 to derive said optimum $h_k(k)$ to minimize~~
15 ~~J and,~~
~~step 4 implements said algorithm, parameters, and stopping rule~~
~~selected in step 3 to derive said optimum $\psi(k)$ to~~
~~minimize J equal to the weighted sum of the norm-squared~~
~~error metrics $J(\text{pass})$, $J(\text{stop})$, $J(\text{dead})$, $J(\text{ISI})$, $J(\text{ACI})$,~~
20 ~~said optimum h_k uses said bw to calculate optimum $\psi(n)$ for~~
~~implementation as said Wavelet FIR digital waveform and~~
~~filter time response.~~
~~use inverse discrete Fourier transform of said $\psi(k)$ to calculate~~
 ~~$\psi(n)$ which minimizes J ,~~
25 ~~use said $\psi(n)$ for the transmitted data symbol waveform in the~~
~~communications transmitter and,~~
~~use complex conjugate of said $\psi(n)$ for the impulse response of~~
~~the detection filter in the communications receiver to~~
~~remove the received $\psi(n)$ and recover said transmitted data~~
30 ~~symbols.~~

Claim 9. (~~newdeleted~~) ~~Wherein said mother Wavelet generates multi-resolution dilated Wavelets, comprising steps and design: said Wavelet parameters are~~

~~a) scaling parameter p dilates sampling by factor 2^p~~

5 ~~equivalent to sub-sampling by factor 2^p ,~~

~~b) translation parameter q translates said ψ by qM digital samples,~~

~~c) frequency offset k is set by design,~~

~~d) symbol repetition interval said M remains constant,~~

10 ~~e) Wavelet length said L in units of said M remains constant,~~

~~step 1 uses said design harmonics ψ_k to generate said FIR time response $\psi(n_p)$ at baseband with said bw equation~~

~~$$\psi(n_p) = \sum_k bw(n_p+1, k+1) \psi_k(k)$$~~

15 ~~recited in claim 7 with~~

~~$$bw = 2 \cos(2\pi n_p k / ML) \text{ for } n_p > 0$$~~

~~wherein $n_p = n / 2^p$ is n sub-sampled or equivalently dilated by the factor 2^p ,~~

~~step 2 uses said $\psi(n_p)$ to construct said multi-resolution~~

20 ~~Wavelet $\psi_{p,q,M,L,k}$ with equation~~

~~$$\psi_{p,q,M,L,k} = 2^{-(p/2)} \psi(n_p - qM) \exp(j2\pi kn_p / ML)$$~~

~~which is said FIR time response for parameters p,q,M,L,k~~

~~wherein the subset p,M,L are the scale parameters,~~

~~design of said multi-resolution Wavelet includes~~

25 ~~f) said T for n is increased to $T2^p$ for n_p ,~~

~~g) said $1/T$ is reduced to $1/T2^p$,~~

~~h) said ψ symbol rate $1/MT$ equal to said channel-to-channel separation is reduced to $1/MT2^p$ in Hz and,~~

~~i) said ψ length $(ML+1)T$ in seconds is stretched to~~

30 ~~$(ML+1)T2^p$ in seconds.~~

Claim 10 (~~new~~currently amended) A method for implementing Wavelet waveforms and filters for multi-resolution communication applications derived from said mother Wavelets in claims 7 or 8, comprising steps:~~Wherein said mother Wavelet generates multi-resolution constant sample rate dilated Wavelets, comprising steps and design:-~~

said mother Wavelet is designed for application to an M channel polyphase filter bank as depicted in claims 7 or 8 wherein M is the spacing between Wavelets within said channels for the Nyquist digital filter bank sample rate $1/T$,

said multi-resolution changes the number of said user channels to $M2^p$ while keeping the same channel filter design which means said Nyquist digital sample rate is changed to $2^p/T$ wherein Wavelet scale parameter p is an integer,

said multi-resolution Wavelet FIR $\psi(n)$ is derived from said mother Wavelet design harmonics $\psi(k)$ using the inverse discrete Fourier transform for the mapping of $\psi(k)$ to $\psi(k)$, use said $\psi(n)$ for the transmitted data symbol waveform for each transmit channel in the communications transmitter and, use complex conjugate of said $\psi(n)$ for the impulse response of the detection filter bank in the communications receiver which is used to remove the received $\psi(n)$ and recover said transmitted data symbols.

~~said Wavelet parameters are~~

~~a) said p dilates said ψ to increase said length from $ML+1$ to M_pL+1 where $M_p=M2^p$ is the dilated interval between contiguous ψ 's,~~

~~b) said q translates said ψ by qM_p digital samples,~~

~~e) said k is set by design,~~

~~d) said $M_p=M2^p$ is dilated M ,~~

~~e) said L remains constant,~~

~~step 1 uses said design harmonics ψ_k to generate said FIR time response $\psi(n_p)$ at baseband with said bw equation~~

~~$$\psi(n) = \sum_k bw(n+1, k+1) \psi_k(k)$$~~

~~recited in claim 7 with~~

~~$$bw = 2 \cos(2\pi nk/M_p L) \text{ for } n > 0,$$~~

~~step 2 uses said $\psi(n)$ to construct said multi-resolution Wavelet~~

5 ~~$\psi_{p,q,M,L,k}$ with equation~~

~~$$\psi_{p,q,M,L,k} = 2^{-(p/2)} \psi(n - qM_p) \exp(j2\pi kn/M_p L)$$~~

~~which is said FIR time response for parameters p, q, M, L, k ,
design of said multi-resolution Wavelet includes~~
~~f) said T remains constant,~~

10 ~~g) said $1/T$ remains constant,~~

~~h) said ψ symbol rate $1/MT$ equal to said channel-to-channel
separation is reduced to $1/M_p T = 1/MT 2^p$ in Hz and,~~
~~i) said ψ length $(ML+1)T$ in seconds is stretched to~~
 ~~$(ML 2^p + 1)T$ in seconds.~~

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Claim 11. (newdeleted) Wherein said mother Wavelet
generates multi-resolution up-sampled Wavelets, comprising steps
and design:

20 ~~said Wavelet parameters are~~

~~a) said p up-samples said digital sampling rate $1/T$ to $2^p/T$,~~
~~b) said q translates said ψ by qM digital samples,~~
~~c) said k is a design parameter,~~
~~d) said M is constant,~~

25 ~~e) said L is constant,~~

~~step 1 uses said design harmonics h_k, ψ_k to generate said FIR~~
~~time response $\psi(n_p)$ at baseband with said equation~~

~~$$\psi(n_p) = \sum_k bw(n_p+1, k+1) \psi_k(k)$$~~

~~recited in claim 7 with~~

30 ~~$bw = 2 \cos(2\pi n_p k/ML)$ for $n_p > 0$~~

~~wherein n_p is n up-sampled by the factor 2^p and defined
by equations~~

~~$$n_p = n_p + n 2^p$$~~

$$n_p = 0, 1, 2, \dots, 2^p - 1$$

~~wherein n_p is the index over the additional samples added to each sample n by said up-sampling,~~

~~step 2 uses said $\psi(n_p)$ to construct said multi-resolution Wavelet~~

5 ~~$\psi_{p,q,M,L,k}$ with equation~~

$$\psi_{p,q,M,L,k} = 2^{-(p/2)} \psi(n_p - qM) \exp(j2\pi kn_p/ML)$$

~~which is said FIR time response for parameters p, q, M, L, k , design of said multi-resolution Wavelet includes~~

~~f) said T is decreased to $T/2^p$,~~

10 ~~g) said $1/T$ is increased to $2^p/T$,~~

~~h) said ψ symbol rate $1/MT$ equal to said channel to channel separation is increased to $2^p/MT$ in Hz and,~~

~~i) said ψ length $(ML+1)T$ in seconds is reduced to~~

~~$(ML+1)T/2^p$ in seconds.~~

15

Claim 12 (~~new~~currently amended) Wherein said multi-resolution Wavelets have properties comprising:

said multi-resolution Wavelets $\psi(n)$ at baseband are derived from

20 said mother Wavelet using said design harmonics $\psi(k)$ and scale parameters said dilation p , said number of samples M over Wavelet spacing, length (L) in units of M , said digital sample rate $1/T$, and translation parameter.

said $\psi(n)$ can be designed to support a bandwidth(B)-time(T)

25 product $BT=1+\alpha$ with no excess bandwidth $\alpha=0$,

~~said scale parameters p, M, L and said design parameter $1/T$ specify~~

~~said multi-resolution Wavelets at baseband and said q, k~~

~~specify time, frequency translations from baseband,~~

~~said design harmonics $\psi_k(k)$ of mother Wavelet are said design~~

30 ~~coordinates for multi-resolution Wavelets,~~

~~said design harmonics $\psi_k(k)$ use said bw matrix to generate said~~

~~multi-resolution Wavelet baseband time response $\psi(n)$ for~~

~~said dilation, dilation of Wavelet length, and up-~~

~~sampling as recited in Claims 9-11 and which is translated in time and frequency to said multi-resolution Wavelet~~

~~$\psi_{p,q,M,L,kT}$~~

~~said design harmonics $\psi_k(k)$ are few in number compared to said~~

5 ~~$\psi(n)$,~~

~~said ψ is designed to support a bandwidth-time product $B_f T = 1 + \alpha$ with no zero excess bandwidth $\alpha = 0$,~~

~~said $\psi(n)$ can be designed to support a bandwidth(B)-time(T) product $BT = 1 + \alpha$ with no excess bandwidth $\alpha = 0$,~~

10 ~~said multi-resolution Wavelets are designed to behave like an accordion in that at different scales said Wavelets are stretched and compressed versions of the mother Wavelet with appropriate time and frequency translation,~~

~~said optimization techniques in claims 7,8 assume said $\psi(n)$~~

15 ~~symmetric about $n=0$ and are applicable to other arrangements of $\psi(n)$ with self-evident modifications,~~

~~optimization algorithms for finding said optimum set of $\psi_k(k)$ use said linear LS waveform and filter design methods recited in claims 7,8 and also use other methods and,~~

20 ~~said linear waveform and filter LS-least-squares design methods can be modified to design waveforms for other applications including bandwidth efficient modulation BEM and synthetic aperture radar-PAR-and,~~

~~other optimization algorithms exist for finding said optimum~~

25 ~~$\psi(n)$.~~